HOSSAM GHANEM

Derivative

The Derivative Of Sum Function

Function	Derivative
c	0
x	1
x^2	$2x$
x^3	$3x^2$
x^n	$n\,x^{n-1}$
$f(x)+g(x)$	$f'(x)+g'(x)$

The derivative by definition

المشتقة العامة

$$f'(x)=\frac{f(x+h)-f(x)}{h}$$

المشتقة عند نقطة

$$f'(a)=\frac{f(x)-f(a)}{x-a}$$

$$D_x[f(x)\cdot g(x)]=f'(x)\cdot g(x)+f(x)g'(x)$$

$$D_x\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)\cdot f'(x)-f(x)g'(x)}{\left(g(x)\right)^2}$$

The Derivative Of Trigonometric Function

Function	Derivative
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\csc x$	$-\csc x\cot x$
$\sec x$	$\sec x\tan x$
$\cot x$	$-\csc^2 x$

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(19) 3.3 *Techniques of differentiation 3.5*

Derivatives of the Trigonometric Functions

Example 1	Find the derivative $f(x) = 2x^3 - 5x^2 + 3x - 11$

Solution

$f'(x) = 6x^2 - 10x + 3$

Example 2	Find the derivative $f(x) = 2x^5 - 4x^{-2} + 3x^{\frac{1}{3}} - 5$

Solution

$f'(x) = 10x + 8x^{-3} + x^{\frac{-2}{3}}$

Example 3	Find the derivative $f(x) = 6x^4 - 5\sqrt{x^3}$

Solution

$f(x) = 6x^4 - 5x^{\frac{3}{2}}$

$f'(x) = 24x - 5 \cdot \frac{3}{2} x^{\frac{1}{2}} = 24x - \frac{15}{2}\sqrt{x}$

Example 4	Find the derivative $f(x) = 2x^{\frac{7}{5}} + 3\sqrt[3]{x^2}$

Solution

$f(x) = 2x^{\frac{7}{5}} + 3\sqrt[3]{x^2}$ $\qquad$ $f'(x) = \frac{14}{5} x^{\frac{2}{5}} + 2x^{\frac{-1}{3}}$

Example 5	Find the derivative $f(x) = (x^3 - 3x^2)(3x^5 - 7)$

Solution

$f'(x) = (6x^2 - 6x)(3x^5 - 7) + (x^3 - 3x^2)(15x^4)$

Example 6	Find the derivative $f(x) = x^{\frac{3}{5}}(5x^2 + 9x - 1)$

Solution

$f'(x) = \frac{3}{5} x^{\frac{-2}{5}}(5x^2 + 9x - 1) + x^{\frac{3}{5}}(10x + 9)$

Example 7	Find the derivative $f(x) = 2x^{\frac{7}{2}}\left(4x^2 + 9\sqrt[4]{x^3} - 1\right)$

Solution

$f'(x) = 7x^{\frac{5}{2}}\left(4x^2 + 9\sqrt[4]{x^3} - 1\right) + 2x^{\frac{7}{2}}\left(8x + 12x^{\frac{1}{3}}\right)$

Example 8	Find the derivative $f(x) = \frac{4x^2 - 3}{5x^5 + 2x}$

Solution

$f'(x) = \frac{(5x^5 + 2x)(8x) - (4x^2 - 3)(25x^4 + 2)}{(5x^5 + 2x)^2}$

Example 9	Find the derivative $f(x)=\dfrac{\sqrt{x}}{2x^3-3x}$

Solution

$$f'(x)=\frac{(2x^3-3x)\cdot\frac{1}{2\sqrt{x}}-\sqrt{x}(6x^2-3)}{(2x^3-3x)^2}$$

Example 10	Find the derivative $f(x)=(4x^3-3x^2)^5$

Solution

$$f'(x)=5(4x^3-3x^2)^4(12x^2-6x)$$

Example 11	Find the derivative $f(x)=(x^2-3)^{\frac{5}{9}}$

Solution

$$f'(x)=\frac{5}{9}(x^2-3)^{\frac{-4}{9}}(2x)$$

Example 12	Find the derivative $f(x)=\sqrt{x^2-7x+3}$

Solution

$$f'(x)=\frac{2x-7}{2\sqrt{x^2-7x+3}}$$

Example 13	Find the derivative $f(x)=7x^2(4x^2-3x)^5$

Solution

$$f'(x)=14x\,(4x^2-3x)^5+35x^2(4x^2-3x)^4(8x-3)$$

Example 14 45 10 May, 2009	Let $f(t)=t^2\ \sin t$. Find $f''(0)$

Solution

$$f'(t)=2t\ \sin t+t^2\ \cos t$$
$$f''(t)=2\sin t+2t\ \cos t+2t\ \cos t-t^2\ \sin t$$
$$f''(0)=0+0+0+0=0$$

Example 15 37 May 4, 2006	Find $f''(1)$, where $f(x)=\dfrac{x^2-1}{x}$ (2 pts.)

Solution

$$f'(x)=\frac{x(2x)-(x^2-1)(1)}{x^2}=\frac{2x^2-x^2+1}{x^2}=\frac{x^2+1}{x^2}$$
$$f''(x)=\frac{x^2(2x)-(x^2+1)(2x)}{x^4}=\frac{-2x}{x^4}=\frac{-2}{x^3}$$
$$f''(1)=\frac{-2}{1}=-2$$

3.5 Derivatives of the Trigonometric Functions

$\frac{d}{dx}(\sin x) = \cos x$	$\frac{d}{dx}(\cos x) = -\sin x$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\frac{d}{dx}(\cot x) = -\csc^2 x$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\frac{d}{dx}(\csc x) = -\csc x \cot x$

Example 1 Find the derivative $f(x) = 4\tan x$

Solution

$f'(x) = 4\sec^2 x$

Example 2 Find the derivative $f(x) = 4\sec x \tan x$

Solution

$f'(x) = 4\sec x \cdot \sec^2 x + 4\sec x \tan x \cdot \tan x$
$= 4\sec^3 x + 4\sec x \tan^2 x$

Example 3 Find the derivative $f(x) = 5x^2 \sin x$

Solution

$f'(x) = 10x \sin x + 5x^2 \cos x$

Example 4 Find the derivative $f(x) = 4x^3 + x^2 \tan x + 7$

Solution

$f'(x) = 12x^2 + 2x \tan x + x^2 \sec^2 x$

Example 5 Find the derivative $f(x) = \frac{1 - \cot x}{2x^2}$

Solution

$f'(x) = \frac{1 - \cot x}{2x^2} = \frac{2x^2(\csc^2 x) - (1 - \cot x)(4x)}{4x^4}$

Example 6 Find the derivative $f(x) = 4\sqrt{x}\csc x$

Solution

$f'(x) = 4 \cdot \frac{1}{2\sqrt{x}} \csc x + 4\sqrt{x}(-\csc x \cot x) = \frac{2}{\sqrt{x}} \csc x - 4\sqrt{x} \csc x \cot x$

Example 7	Find the derivative $f(x)=\dfrac{1-\cos x}{2\sin x}$

Solution

$$f'(x)=\frac{2\sin x\,(\sin x)-(1-\cos x)\cdot 2\cos x}{4\sin^2 x}$$

Example 8	Find the derivative $f(x)=\dfrac{1}{2\cos x\cot x}$

Solution

$$f(x)=\frac{1}{2}\sec x\tan x$$

$$f'(x)=\frac{1}{2}\sec x\tan x\cdot\tan x+\frac{1}{2}\sec x\cdot\sec^2 x$$

Example 9	Find the derivative $f(x)=\csc x\sin x$

Solution

$$f'(x)=-\csc x\cot x\sin x+\csc x\cos x$$

Example 10	Find the derivative $f(x)=\dfrac{1+\sec x}{2-\csc x}$

Solution

$$f'(x)=\frac{(2-\csc x)(\sec x\tan x)-(1+\sec x)(\csc x\cot x)}{(2-\csc x)^2}$$

Example 11	Find the derivative $f(x)=\dfrac{1+\sec x}{\tan x+\sin x}$

Solution

$$f'(x)=\frac{(\tan x+\sin x)(\sec x\tan x)-(1+\sec x)(\sec^2 x\cos x)}{(\tan x+\sin x)^2}$$

Example 12	Find $f'(x)$, where $f(x)=\dfrac{\sec x}{x^3+\cot x}$

Solution

$$f'(x)=\frac{(x^3+\cot x)(\sec x\tan x)-\sec x\,(3x^2-\csc^2 x)}{(x^3+\cot x)^2}$$

Example 13	Find $f'(x)$, where $f(x)=\dfrac{x^2\sin x}{1+\cos x}$

Solution

$$f'(x)=\frac{(1+\cos x)(2x\sin x+x^2\cos x)-x^2\sin x\,(-\sin x)}{(1+\cos x)^2}$$

Example 14 42 March 29, 2006	Given $f(x) = x^2 + x\cos^2 x - 1$. Use the intermediate value theorem to show that there is a real number c between $-\frac{\pi}{2}$ and 0 such $f'(c) = 0$.

Solution

$f(x) = x^2 + x\cos^2 x - 1$

$f'(x) = 2x + \cos^2 x + x \cdot 2\cos x\,(-\sin x)$

$f'(0) = 0 + 1 + 0 = 1 > 0$

$f'\left(-\frac{\pi}{2}\right) = -\pi + 0 - \pi(0)(-1) = -\pi < 0$

$\therefore \exists\, c \in \left(-\frac{\pi}{2}, 0\right)$ such that $f'(c) = 0$ "$I.V.T$"

Homework

1 55 April 8, 2010	(3pts) Determine $f'(\pi)$, for $f(x) = \sin(x)\tan(x) + \dfrac{3\cos(x)}{1+\sin(x)}$
2 59 9 July 2011	Question 1. [4+2+2 pts.]: Let $g(x) = \begin{cases} (2x+1)\cos x & \text{if } x \le 0 \\ \dfrac{1-\sqrt{x+1}}{x} & \text{if } x > 0 \end{cases}$ (a) Show that g discontinuous at $x = 0$ (b) Use part (a) to determine if g is differentiable at $x = 0$ (c) Find $g'(x)$, if $x < 0$